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ALGORITHM OF THE ZONAL SOLUTION OF RADIATION—CONDUCTION HEAT-TRANSFER PROBLEMS

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UDC 536.3

A numerical method is proposed to compute the stationary radiation—conduction heat transfer in semitransparent materials on the basis of a zonal approach.

The development of methods to compute the radiation—conduction heat transfer [1] is of great value for many thermal-engineering applications. The use of high-speed electronic computers with sufficient mathematical support permits the execution of a penetrating computational theoretical analysis of this kind of heat transfer in absorbing inhomogeneous media with a detailed accounting of the frequency—temperature dependence of the optical characteristics in both the bulk and on the boundaries of the radiating system [2-5]. Great attention is paid to overcoming the mathematical difficulties in solving radiation—conduction heat-transfer (RCT) problems in the presence of semiopacity of the boundary surfaces [4, 5], as well as moving phase interfaces [6].

It should be noted, however, that the high level of detail achieved in computations in [2-6] is as yet realized for the one-dimensional plane-parallel case. Nevertheless, the need to produce computational methods permitting the analysis of RCT in two- and three-dimensional systems of different configuration is already overdue. Hence, by taking into account the difficulties of realizing exact formulations of complex heat-transfer problems for arbitrary volume geometry conditions, the prospects of approximate zonal methods [1] based on the approximation of the initial radiation integral equations by a system of algebraic equations [7] are noted. Meanwhile, the inadequately extensive utilization of these methods in the theory of complex heat transfer is indicated in [1]. An analysis of foreign investigations of the application of approximate methods of solving complex heat-transfer problems in bulk systems is presented in [8], and reduces to recommendations to utilize the so-called method of generalized angular coefficients in the RCT domain in [8]. The expediency of using the statistical testing (Monte Carlo) method, whose efficiency is demonstrated in a number of examples, is indicated in [8] for the determination of the generalized angular coefficients as well as the radiation exchange coefficients for the solution of different complex heat-transfer problems. In particular, the simplicity and physical nature of the solution of problems with complex bulk geometries of the radiating systems by the method mentioned are noted. The prospects of utilizing the Monte Carlo method to model radiation transport processes are also noted in [1].

The results of trying out the algorithm for the approximate solution of RCT problems on the basis of a zonal approach [7, 9] and the utilization of the Monte Carlo method to determine the radiation exchange coefficients [10, 11], as well as a finite-difference scheme to take account of heat transfer by heat conduction [12] are presented in this paper. The

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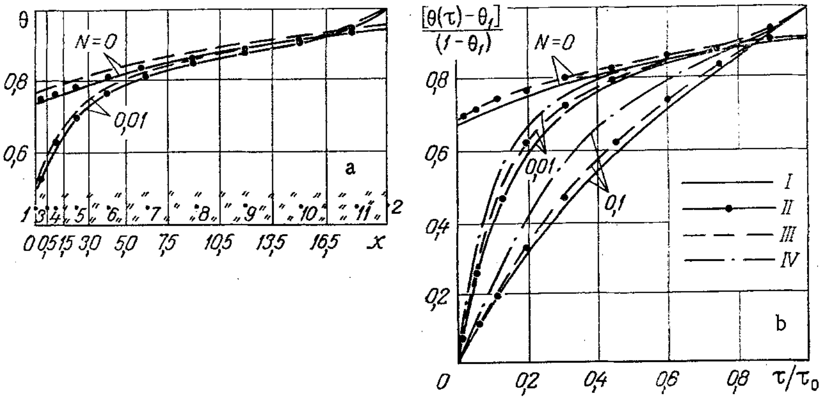

Fig. 1. Dimensionless temperature distributions in a plane layer of absorbing and radiating media under combined heat transport by heat conduction and radiation ($\tau_0 = 1.0$; $\varepsilon_1 = \varepsilon_2 = 1.0$): a) $\theta_1 = 0.5$, $\theta_2 = 1.0$; b) $\theta_1 = 0$, $\theta_2 = 1.0$. I) Exact computation [8, 14]; II) zonal solution for 3000 tests; III) zonal solution for 1000 tests; IV) approximate method of generalized angular radiation coefficients [8].

distinguishing feature of the algorithm developed is the assurance of sufficient generality in modeling the RCT in two- and three-dimensional systems of arbitrary geometry, with selectivity and radiation scattering as well as the inhomogeneities in the optical and thermophysical characteristics taken into account.

The equation of energy conservation under the combined effect of heat conduction and radiation in an absorbing (radiating) medium and heat-transfer stationarity conditions can be written as follows:

$$\operatorname{div}[\lambda(r,\ T)\operatorname{grad} T(r)] - \operatorname{div}\bar{q}_R + h(r) = 0. \qquad (1)$$

The algorithm proposed for the solution of this equation for systems with bulk geometry can be separated into four steps.

1. In the first step, the generalized angular coefficients between all the computational sections isolated in the system, both bulk and surface, are evaluated by the Monte Carlo method for absorption coefficients (or attenuation factors in the case of a scattering medium) given in the medium. Finite volumes are the computational sections in the bulk of the model in the case of utilizing the Monte Carlo method, with partition of the space into regions [10], or the nodes of the finite-difference mesh directly for a nodal partition and nonlinear geometry [11]. It should be noted that in the case of the partition into regions, when the mid-zonal generalized angular coefficients are evaluated, the nodes of the finite-difference mesh are placed at the centers of gravity of the geometric figures forming the bulk and surface zones.

2. Later, in conformity with the assumption about diffuseness of the radiation and reflection by surfaces as well as about the isotropy of scattering in the bulk during the passage from the generalized to the constitutive angular coefficients, solution of a system of linear algebraic equations is used [9, 13]. The solution of the system of linear equations in this algorithm is by an iteration method for which the initial approximation is a matrix of generalized angular coefficients.

When using a three-dimensional matrix of dimension $z \times m \times m$ of selective constitutive angular coefficients, the divergence $\bar{q}_R$ for the i-th volume can be written as follows

$$(\operatorname{div}\bar{q}_R)_i = -\frac{1}{V_i}\sum_{j=1}^{m}\sum_{k=1}^{z} 4K_{ni}^{\Delta\lambda_k} V_i f_{ij}^{\Delta\lambda_k}\left(\pi\int_{\Delta\lambda_k} I_{0i}^{\lambda}d\lambda - \pi\int_{\Delta\lambda_k} I_{0j}d\lambda\right). \qquad (2)$$

Using the concept of total radiation exchange coefficients with respect to the spectrum a^{Σ}_{ij} [9], expression (2) can be represented in an abbreviated form more convenient for calculational procedures

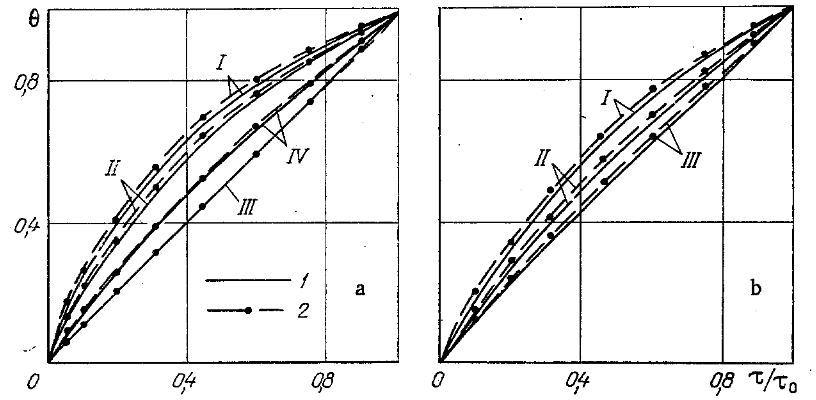

Fig. 2. Influence of radiation scattering (a: N = 0.5; $\varepsilon_1 = \varepsilon_2 = 1.0$) and emissivity of the boundary surfaces (b: N = 0.1, Sc = 0.5) on the temperature distribution in the layer: 1) Exact computation [14]; 2) zonal solution. For a: I) Sc = 0; II) 0.5; III) 1; IV) 0.9; b: I) $\varepsilon_1 = 0$, $\varepsilon_2 = 1$; II) $\varepsilon_1 = \varepsilon_2 = 0.5$; III) $\varepsilon_1 = \varepsilon_2 = 0.1$.

$$(\operatorname{div} \bar{q}_R)_i = -\sum_{j=1}^{m} a_{ij}^{\Sigma} T_j^4, \; i = 1, \ldots, m_1, \tag{3}$$

where

$$a_{ij}^{\Sigma} = 4\sigma_0 \sum_{k=1}^{z} K_{\mathrm{n}i}^{\Delta\lambda_k} f_{ij}^{\Delta\lambda_k} \alpha_i^{\Delta\lambda_k}; \tag{4}$$

$$a_{ii}^{\Sigma} = 4\sigma_0 \sum_{k=1}^{z} K_{\mathrm{n}i}^{\Delta\lambda_k} (f_{ii}^{\Delta\lambda_k} - 1)\, \alpha_i^{\Delta\lambda_k}; \tag{5}$$

$$\alpha_i^{\Delta\lambda_k} = \pi \int_{\Delta\lambda_k} I_{0i}^{\lambda} d\lambda / \sigma_0 T_i^4 . \tag{6}$$

3. In the next step, the matrices of the conductive heat transfer coefficients are formed which can easily be used to calculate the divergence of the heat-conduction flux vector in any bulk node of the computational model.

By using numerical differentiation based on the method of finite differences [12], both the first derivative of the temperature with respect to the coordinate (the heat flux) and the second derivative (the divergence of the heat flux) can be expressed in the form of an algebraic sum of products of the appropriate differentiation coefficients by the value of the temperature of nodes surrounding the node i. Thus, for instance, for the one-dimensional case

$$\{\operatorname{div}[\lambda(x, T)\operatorname{grad} T(x)]\} = \left[\frac{d}{dx}\left(\lambda_i \frac{dT}{dx}\right)\right] = \sum_{j=l}^{n} b_{ij}'' T_j. \tag{7}$$

Here l, n is the node number taking into account that i is in the range (l, n).

4. Substituting (3) and (7) into the initial energy equation (1) for a stationary heat-transfer process, we arrive at a nonlinear algebraic equation with a free term. On the whole, the heat-transfer model is a system of m nonlinear thermal balance and heat-transmission equations of the following kind:

for the bulk zone i

$$\sum_{j=1}^{m} a_{ij}^{\Sigma} T_j^4 + \sum_{j=l}^{n} b_{ij}'' T_j + h_i = 0, \; i = 1, \ldots, m_1; \tag{8}$$

for the surface zone, the heat flux balance is considered

$$\sum_{j=1}^{m} A_{ij}^{\Sigma} T_j^4 + \sum_{j=l}^{n} b_{ij}' T_j + q_i = 0, \; i = m_1 + 1, \ldots, m. \tag{9}$$

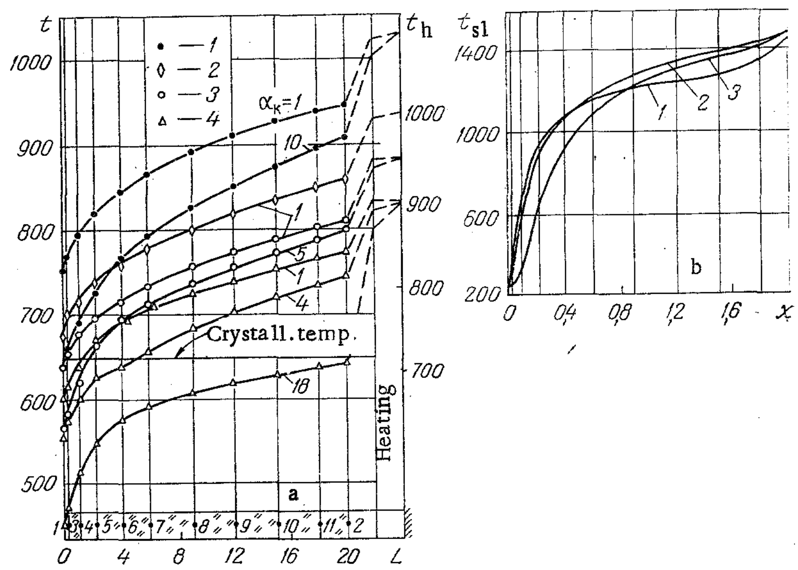

Fig. 3. Temperature distributions in a layer of semitransparent material: a) crystallization of a NaI melt for different heater temperature levels (t_h, °C) and blowing intensity (α_k, W/m²·K): 1) $t_h = 1100$°C; 2) 1000; 3) 950; 4) 900; b) synthetic slag used in CICM: 1) Slag 1; 2) slag 2; 3) slag 3. L, m·10^{-2}; t_{sl}, °C; x, m·10^{-3}.

Here A_{ij}^{Σ} are the radiation exchange factors for the surface zones

$$A_{ij}^{\Sigma} = \sigma_0 \sum_{k=1}^{z} \varepsilon_i^{\Delta\lambda_k} f_{ij}^{\Delta\lambda_k} \alpha_j^{\Delta\lambda_k} ; \tag{10}$$

$$A_{ii}^{\Sigma} = \sigma_0 \sum_{k=1}^{z} \varepsilon_i \left(f_{ii}^{\Delta\lambda_k} - 1\right) \alpha_i^{\Delta\lambda_k} . \tag{11}$$

Let us note that (9) for the surface zones in contact with a semiopaque medium is a record of the boundary conditions for the solution of the energy equations (1) that are formed during the solution of the system of nonlinear equations (8) and (9). The boundary temperature values, therefore, taken as unknown in the conditions of the problem, permit extension of the analysis when investigating the influence of the external effects (irradiation, blowing, etc.). Taking convection into account is by including the convective heat-elimination factor in the appropriate linear terms in the summation in (8) and (9). The numerical solution of the system of nonlinear equations (8) and (9) is easily realized on an electronic computer by using a rapidly convergent iteration procedure based on the Newton method [8].

To test the proposed algorithm, the results of computations by this method for fixed values of the boundary shell parameters were compared with the data of exact numerical solutions of Viscanta and Grosch as well as Lee and Ozisik presented in [8, 14]. The computations were performed for a different number of tests (1000 and 3000 per zone), as well as in a broad range of variation of the radiation and conductive properties of the medium. The comparison displayed good agreement between the results obtained by the exact and the described approximate methods. Thus, known curves of the relative temperature distribution $\theta(\tau)$ in a layer are represented in Fig. 1a for pure radiation ($N = 0$) and the case of radiation interaction with heat conduction ($N = 0.01$) [14], on which the temperature values obtained are superposed at the nodes of a finite-difference mesh, which can be estimated by partitioning the x axis into computational sections. It is seen from Fig. 1a that the accuracy of the proposed method is determined mainly by the accuracy in computing the radiation component $(\mathrm{div}\ \bar{q}_R)_i$, whose numerical value depends on the size of the random sample because of the statistical nature of the calculation. Thus, e.g., a threefold increase in the number of tests would permit a reduction in the error of calculating the local temperature in the layer from 5% for given values of the governing parameters to errors not exceeding 1% (Fig. 1a). The computation time for this on a low-speed machine (Minsk-22M) in combination with the time expended in solving the system of nonlinear equations did not exceed 1 h.

It should be noted that the inaccuracy in the matrix of the generalized angular coefficients obtained directly by the Monte Carlo method is a source of error in all the further computations as the values of the parameters being given vary (the wall temperature relationship, the heat conductivity of the medium, the radiation characteristics of the surfaces and bulk). However, a further increase in the number of tests did not result in any refinement of the results because of the limited possibilities being used in the program of the pseudo-random number transducers.

A comparison between the proposed algorithm and other approximate methods of computing the temperature fields in a layer of a radiation heat-conducting medium is of interest. Results of calculations by Howell when using the so-called method of generalized angular coefficients [8] are superposed in Fig. 1b in addition to the curves obtained by the exact and zonal solutions. As is seen from the figure, the Howell method is more approximate than the method proposed. The circumstance that radiation terms governing the mutual transport of radiation energy between volumes of the medium with unknown temperatures (according to the formulation of the problem) are not used in writing the energy equation for the volume element of the medium is the basis of the error in the Howell computational scheme. The influence of these radiation terms in the Howell method is in the fractions of effective radiation flux from the surfaces that take account of radiation energy transfer between volume elements of the medium, however, in conformity with only the temperature profiles obtained in the absence of heat conductivity ($N = 0$) [8]. A result is the chronic exaggeration of the temperature on the layer for $N > 0$ (Fig. 1b).

The possibility of obtaining results with acceptable accuracy is shown in Fig. 2 for the case of a dissipative medium and existing substantial radiation reflection at the boundaries. As computations showed, the error in determining the temperature fields in a layer with the boundary temperature relationships $\theta_1 = 0$ and $\theta_2 = 1$ diminishes as the dissipative properties of the medium increase (Fig. 2a). This is a completely evident fact since the radiation ceases to influence the temperature distribution which is here linear in form as in the case of just heat conduction ($N = \infty$), as the Schuster criterion approaches one (purely dissipative medium). Upon introducing diffuse reflection on the boundaries into the computation, the error of the calculation increases as the surface emissivity increases (Fig. 2b). This can be explained by the fact that as the constitutive angular coefficients are being found by solving a system of linear equations, the inaccuracy contained in the initial matrix of generalized angular coefficients obtained by the statistical testing method accumulates.

Therefore, the computations performed showed that the zonal approach used in the presented algorithm to solve RCT problems permits results to be obtained for a sufficiently rough partition of the space into computational sections, that will agree satisfactorily with the exact solutions. The reserve in the rise in accuracy is the perfection of the procedure to determine the generalized angular coefficients in order to reduce them in complete conformity with the reciprocity law.

In thermal-engineering applications, the accuracy attained in this paper in calculating the temperature fields (Figs. 1 and 2) can satisfy the demands of engineering computations. Sufficiently rough assumptions are frequently made in these latter, which are related either to the complete transparency taken for the material or to the unjustified utilization of a diffusion approximation model for low and medium optical density of the medium. Presented below as an illustration are certain results of solving two applied problems, which are obtained by using the zonal approximation algorithm presented above. In particular, the problem of computing the temperature conditions for the process of growing alkali-halide single crystals from a NaI melt by the method of the moving isotherm in shaft resistance furnaces is considered [15].*

The crystallization process occurs over a sufficiently long time in electric furnaces. The temperature field governs the rate of crystal growth and its quality. In this

*The recommendations of Dr. G. K. Rubin and Kand. V. Ya. Lipov (All-Union Scientific-Research Institute of Electrothermal Equipment) were used in compiling the mathematical model and formulating the problem.

connection, the technological requirements on the crystallization regime are sufficiently high. The temperature field is controlled during crystal growth by changing the temperature of the heater and the consumption of the cooling gas. In this paper, the influence of the mentioned effects on the temperature field formation in a semitransparent material in order to make a well-founded selection of the furnace temperature regimes is investigated by using a one-dimensional mathematical model. Certain curves of the temperature state of the material in the crystallization stage are presented in Fig. 3a. The values of the control parameters, the heater temperature t_h and the consumption of the cooling gas expressed in terms of the convective heat elimination factor α_c, are also presented in Fig. 3a. The temperature of the lower shell (refrigerator) was 50°C here in all the modifications.

An interesting phenomenon was detected as a result of the computations, viz., that the temperature gradient in the crystallization zone increases as a small solid phase layer appears (curves $\alpha_c = 1$ and 5 W/m²·K at $t_h = 950$°C, Fig. 3a). This can be explained by the increase in radiation heat conduction in the zone of less optically dense solidifying material, and as a result, its more intense cooling. Another interesting fact of the material cooling process was that the efficiency of the influence of the blowing on the formation of a definite temperature gradient was reduced somewhat at the temperature drop diminished between the refrigerator and the heater (Fig. 3a). In connection with the phenomena noted, some recommendations can be given on controlling the thermal mode of the crystallization process. In particular, at the time of the appearance of the solid phase it is expedient to have a sufficient consumption of cooling gas so as to maintain the reduction in blowing intensity efficiently at the necessary level to increase the temperature gradient in this zone at the initial instant of the crystallization. The heater temperature later diminishes sufficiently smoothly with the appearance of a significant solid phase layer (30-40% of the whole layer thickness), keeping the cooling gas consumption negligible.

The algorithm of the zonal solution of RCT problems was also used to compute the temperature field in slags used in the continuous ingot casting machine (CICM) crystallizers. The problem of selecting the best slag composition to assure the necessary heat insulating and lubricating properties by their thermophysical and radiation characteristics was posed.

At this time the slags used in CICM are opaque and the heat transfer therein is considered with the effective coefficient of heat conduction taken into account. Temperature dependences of the bulk absorption coefficient were constructed according to the relationship [8, 14]

$$K_{ai} = \frac{16\,\sigma_0 T_i^3}{3\,\lambda_R} \tag{12}$$

in computations by means of the proposed algorithm on the basis of data about the effective and radiation heat conductivity.*

A one-dimensional RCT model in a 0.002-m-thick layer with a characteristic bunching of the finite-difference mesh in the gradient region (near the crystallizer wall being cooled, Fig. 3b) was used for the analysis. The temperature of the ingot surface was fixed (hot wall, $T_{in} = 1743$°K) as was the temperature of the cooling water ($T_W = 313$°K). The temperature of the inner crystallizer surface was desired and depended, as did the temperature distribution in the layer, on the optical and thermophysical properties of the slag. The emissivities of the ingot and crystallizer surfaces were taken at 0.8 and 0.7, respectively.

The temperature distribution curves in the slag are presented in Fig. 3b. Slag 1 is transparent in the whole temperature range. As the temperature grows the λ_R for this slag rises substantially, specifying a relatively high "radiation" conductivity in the temperature domain exceeding 1100°C (Fig. 3b). The coefficient of molecular heat conductivity in the temperature band under investigation varies negligibly (from 2 to 4 W/m·K) almost linearly. Slag 2 becomes negligibly transparent at temperatures above 1100°C, and the coefficient of molecular heat conductivity has no monotonically rising temperature dependence. At low temperatures it exceeds the corresponding coefficient for slag 1 slightly, while it is somewhat lower in the high temperature domain. This is due to its good insulating properties in layers adjoining the ingot surface. Slag 3 is analogous to slag 2 in its radiation heat-conductivity component. However, it has a high molecular heat conductivity in the medium

*The experimental data of M. V. Frolov (Steel metallurgy department of the Ural Polytechnical Institute) were used.

and low temperature domains (below 800°C). This explains the specific nature of the temperature curve in the gradient domain (Fig. 3b).

On the whole, the analysis of the heat transfer through a slag layer permitted the clarification of the advantages of slag 2, which assures a reduction in the forces directed at dragging the metal from the crystallizer because of the most stable liquid phase layer in the area of contact with the ingot. At the same time, this slag yields a certain reduction in the heat flux to the crystallizer wall, which explains the lower heat losses.

NOTATION

λ, coefficient of heat conductivity; r, a coordinate; T, temperature; $\bar{q}_R$, radiation heat flux vector; h, bulk power of the internal energy sources; z, number of spectral ranges; m_1, number of bulk zones; m, total number of bulk and surface zones; f_{ij}, referred constitutive angular coefficient; K_a, absorption coefficient; $\Delta\lambda_k$, k-th spectral band; V, volume; $I_{0\lambda}$, spectral intensity of absolutely black radiation; a^{Σ}_{ij}, total radiation transfer coefficient in the spectrum; σ_0, Stefan–Boltzmann constant; b'_{ij} and b''_{ij}, coefficients of single and double differentiation, respectively, of the temperature with respect to the coordinate; ε, surface emissivity; q_i, quantity of heat transferred from zones with known temperature; N, radiation conduction parameter; θ, dimensionless temperature; τ_0, optical thickness of the layer; Sc, Schuster criterion; λ_R, coefficient of radiation heat conductivity.

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TEMPERATURE FIELD OF LAMINAR-INHOMOGENEOUS BEDS

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Results are presented of a theoretical investigation of the temperature field of oil beds with a nonuniform structure, with applications in the technology of selective thermoinjection.

Heat transfer accompanying the filtration of liquid in permeable media is the physical basis of many processes in mining, the power industry, and chemical engineering.

The formulation of multidimensional problems is a particularly urgent matter. Such problems include those concerning filtration in an infinite porous medium with point sources and sinks and in a plane porous bed with heat transfer to the roof and floor; problems for collectors of different form, taking free convection into account; etc.

A possible approach to the solution of such problems consists in the use of the idea of "homogenization" of the heterogeneous medium [1], analogously to the methods of the mechanics of interpenetrating media [2] or in the approximation of instantaneous temperature equalization of the two phases [3]. On this basis, fairly many problems may be solved, but in practice they give rise to a series of serious objections. For example, one unsolved problem is the choice of the heat-transfer coefficient at the interface of the two phases, since this is not simply the heat-transfer coefficient between the individual elements of the filling and the liquid, but a coefficient or function which must take into account all the arbitrariness of the given approach. In addition, the transfer coefficients in the equations are not actually the weighted means of the molecular coefficients, as concluded, for example, in [4, 5].

Processes of nonsteady heat transfer accompanying one-dimensional filtration in fillers consisting of small particles with low thermal resistance are usually described using the formulation of the problem first proposed in [6, 7]. However, the results obtained are difficult to use in developing engineering methods of calculation for the heat transfer in more complex multidimensional filtration regions.

In [8], the problem of describing the heat conduction for an analogous physical situation was considered in more detail on the basis of a generalized equation for one dependent variable obtained in [8].

In mining thermophysics (in developing methods of creating systems for the extraction of petrogeothermal resources, and also thermal methods of treating petroleum beds, etc.), it is necessary to develop a method of calculating the heat transfer accompanying filtration in collectors of complex geometry with large structural elements. In constructing the model in this case, it is more correct to use a formulation of the problem in which the finite heat conduction of the elements of the permeable bed is taken into account (see, e.g., [9-11]). A more detailed review of methods of calculating the nonsteady heat transfer accompanying one-dimensional filtration is given in [12].

The use of accurate solutions of heat-conduction problems for particles of the bed and the surrounding rock mass reduces the system of energy equations to an integrodifferential equation, which is not readily generalized to the case of multidimensional filtration. Therefore, in [12], a new approach to the solution of problems of this type was proposed.

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